DYNAMIC BEHAVIOUR OF AN EXTRACTION COLUMN WITH STATIONARY CONTINUOUS PHASE. MOMENTS OF THE RESPONSE TO SOLUTE INJECTION

Jana WICHTERLOVÁ

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences, 165 02 Prague 6-Suchdol

Received May 29, 1990
Accepted September 23, 1990

Relations were derived for calculating the volumetric mass transfer coefficient from the moments of concentration response to the solute injection for the stagewise model with back flows and stationary continuous phase.

When analysing the present or designing a new extraction column, we need its mathematical model. With regard to the insufficient knowledge of behaviour of dispersions, we do not manage without measuring on a pilot plant when seeking the model and its parameters.

For investigating and predicting the mass transfer in an extraction column, Wichterlová, Rod and Hančil[1] propose a method which employs the concentration response to the injection of tracer and solute into the apparatus operating with stationary continuous phase, to determine the model parameters. At the next step, the operating conditions are chosen so that the validity of the parameter values obtained is to be expected even for the countercurrent arrangement. By solving the model for these operating conditions and parameters obtained, the concentration profiles and the column efficiency can then be calculated.

The parameter of longitudial mixing of continuous phase is determined from the response to the injection of tracer into this phase in the middle of the column by the method of Pekovich and Pebalk[2]. The responses to the injection of components into the dispersed phase inlet are measured at this phase outlet from the efficient part of column. The dispersed phase hold-up, parameter of longitudial mixing and the volumetric mass transfer coefficient (i.e., the product of mass transfer coefficient and interfacial area) are determined by an iterative method so as to achieve the satisfactory agreement of the first moments and dimensionless variances of calculated and experimental concentration responses in dispersed phase.

The determination of model parameters is facilitated if their good initial estimates are available. These can be provided by the analytical solution of linear model.

The analytical solution of the model with short-cut flow in the dispersed phase and back flow in the continuous phase is reported in paper[1]. For the description of behaviour of mechanically agitated extraction columns, the stagewise model with back flows in both phases is often used. The aim of this work is to obtain the analytical solution of the model with back flows in the form of the dependence of the first moment and dimensionless variance of concentration response in the dispersed phase outlet on hold-up, coefficients of back flow and volumetric mass transfer coefficient.

THEORETICAL

Model with Back Flows

The stagewise model with back flows describing the behaviour of mechanically agitated extraction column with stationary continuous phase (Fig. 1) was derived on the following assumptions:

- the column consists of n ideally agitated stages of the same volume; there exist back flows of phases between neighbouring stages,
- mass transfer occurs only in the stages,
- the flow rate of dispersed phase, Q_x, back flows, q_x, q_y, volumes of phases v_x, v_y, in stages, and volumetric mass transfer coefficient, k_xA, are constant along the column,
- the equilibrium can be approximated by a linear relation, i.e., distribution coefficient m is constant,
- the continuous phase does not flow through the column,

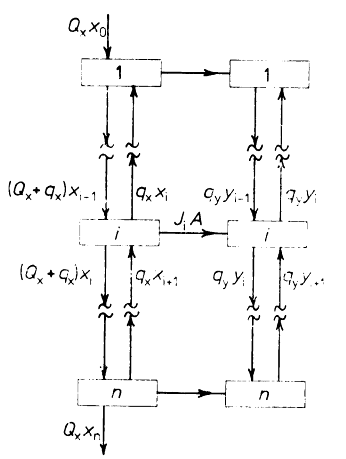

FIG. 1

Stagewise model with back flows in both phases

– the equipment operates in the steady-state hydrodynamic regime without mass transfer before carrying out the injection.

Without loss of generality, it was assumed that

– the column does not contain the observed component before carrying out the injection,

– the injection of M kmol of solute is carried out at time $t = 0$ into the dispersed phase entering the column and is carried out in such a way that it can be approximated by the Dirac pulse $\delta(t)$.

Then the balance equation holds for the solute (y denotes the continuous phase):

$$v_x \frac{dx_1}{dt} = Q_x(x_0 - x_1) + q_x(x_2 - x_1) - J_1 A \,, \tag{1}$$

$$v_y \frac{dy_1}{dt} = q_y(y_2 - y_1) + J_1 A \,, \tag{2}$$

$$v_x \frac{dx_i}{dt} = Q_x(x_{i-1} - x_i) + q_x(x_{i-1} - 2x_i + x_{i+1}) - J_i A \,, \quad i = 2, \ldots, n - 1 \tag{3}$$

$$v_y \frac{dy_i}{dt} = q_y(y_{i-1} - 2y_i + y_{i+1}) + J_i A \,, \tag{4}$$

$$v_x \frac{dx_n}{dt} = Q_x(x_{n-1} - x_n) + q_x(x_{n-1} - x_n) - J_n A \,, \tag{5}$$

$$v_y \frac{dy_n}{dt} = q_y(y_{n-1} - y_n) + J_n A \tag{6}$$

with boundary conditions

$$x_i(0) = 0 \,, \quad i = 1, \ldots n \tag{7}$$
$$y_i(0) = 0 \,,$$

$$x_0(t) = \frac{M}{Q_x} \delta(t) \,, \tag{8}$$

where

$$J_i A = k_x A \left(x_i - \frac{y_i}{m} \right) , \quad i = 1, \ldots n \,. \tag{9}$$

On introducing new quantities T, E, N_x, a, b using Eqs (10)–(13), i.e.

$$T = \Phi \frac{V}{Q_x}, \tag{10}$$

$$E = m \frac{v_y}{v_x} = m \frac{1 - \Phi}{\Phi}, \tag{11}$$

$$N_x = \frac{k_x A}{Q_x}, \tag{12}$$

$$a = \frac{q_x}{Q_x}, \quad b = \frac{q_y}{Q_x}, \tag{13}$$

(where T is the mean residence time of dispersed phase in a stage, $V = v_x + v_y$ is the stage volume, Φ is the hold-up of dispersed phase, E is the extraction ratio, i.e. the ratio of solute amount in the continuous and dispersed phase in a stage at equilibrium, N_x is the mass transfer number, and a, b are coefficients of back flow related to the flow rate of dispersed phase), Eqs (*1*)–(*6*) and (*9*) can be written in the form

$$T \frac{dx_1}{dt} = a(x_2 - x_1) + x_0 + \frac{N_x}{m} y_1 - (1 + N_x) x_1, \tag{14}$$

$$T \frac{E}{m} \frac{dy_1}{dt} = b(y_2 - y_1) + N_x x_1 - \frac{N_x}{m} y_1, \tag{15}$$

$$T \frac{dx_i}{dt} = a(x_{i+1} - 2x_i + x_{i-1}) + x_{i-1} + \frac{N_x}{m} y_i - (1 + N_x) x_i,$$
$$i = 2, \ldots n - 1 \tag{16}$$

$$T \frac{E}{m} \frac{dy_i}{dt} = b(y_{i+1} - 2y_i + y_{i-1}) + N_x x_i - \frac{N_x}{m} y_i, \tag{17}$$

$$T \frac{dx_n}{dt} = a(-x_n + x_{n-1}) + x_{n-1} + \frac{N_x}{m} y_n - (1 + N_x) x_n, \tag{18}$$

$$T \frac{E}{m} \frac{dy_n}{dt} = b(-y_n + y_{n-1}) + N_x x_n - \frac{N_x}{m} y_n. \tag{19}$$

Difference Equations for Moments of Normalized Responses

We are interested in solving the set of equations (*14*)–(*19*) in the form of dependence of the first moment and dimensionless variance of concentration response in the dispersed phase in the n-th stage on parameters N_x, a, Φ.

The r-th moments of normalized responses in the i-th stage are defined as

$$X_i^{(r)} = \frac{Q_x}{M}\int_0^\infty t^r x_i(t)\,dt\,,\quad Y_i^{(r)} = \frac{Q_x}{M}\int_0^\infty t^r y_i(t)\,dt\,. \tag{20}$$

With regard to conditions (7) and to the exponential decrease of functions $x_i(t)$, $y_i(t)$ to zero for $t \to \infty$, we obtain, by integrating per partes, the following relations

$$\frac{Q_x}{M}\int_0^\infty t^r \frac{dx_i}{dt}\,dt = -rX_i^{(r-1)}\,,\quad \frac{Q_x}{M}\int_0^\infty t^r \frac{dy_i}{dt}\,dt = -rY_i^{(r-1)}\,. \tag{21}$$

By multiplying Eqs (14)–(19) by the combination of quantities $t^r Q_x/M$, integrating from $t = 0$ to $t = \infty$ and rearranging, we obtain the set of difference equations for the r-th moments, $X_i^{(r)}$

$$-TrX_1^{(r-1)} = a(X_2^{(r)} - X_1^{(r)}) + X_0^{(r)} + \frac{N_x}{m}Y_1^{(r)} - (1 + N_x)X_1^{(r)} \tag{22}$$

$$-T\frac{E}{m}rY_1^{(r-1)} = b(Y_2^{(r)} - Y_1^{(r)}) + N_x X_1^{(r)} - \frac{N_x}{m}Y_1^{(r)} \tag{23}$$

$$-TrX_i^{(r-1)} = a(X_{i+1}^{(r)} - 2X_i^{(r)} + X_{i-1}^{(r)}) + X_{i-1}^{(r)} + \frac{N_x}{m}Y_i^{(r)} -$$

$$- (1 + N_x)X_i^{(r)}\,,\quad i = 2, \dots n - 1 \tag{24}$$

$$-T\frac{E}{m}rY_i^{(r-1)} = b(Y_{i+1}^{(r)} - 2Y_i^{(r)} + Y_{i-1}^{(r)}) + N_x X_i^{(r)} - \frac{N_x}{m}Y_i^{(r)} \tag{25}$$

$$-TrX_n^{(r-1)} = a(-X_n^{(r)} + X_{n-1}^{(r)}) + X_{n-1}^{(r)} + \frac{N_x}{m}Y_n^{(r)} - (1 + N_x)X_n^{(r)} \tag{26}$$

$$-T\frac{E}{m}rY_n^{(r-1)} = b(-Y_n^{(r)} + Y_{n-1}^{(r)}) + X_n^{(r)} - \frac{N_x}{m}Y_n^{(r)} \tag{27}$$

Since the injection is considered in the form of the Dirac pulse, the following relations hold

$$X_0^{(0)} = 1 \tag{28}$$

$$X_0^{(r)} = 0 \quad \text{for} \quad r \geqq 1\,. \tag{29}$$

With regard to the validity of

$$\int_0^\infty \frac{dx_i}{dt}\,dt = \int_0^\infty \frac{dy_i}{dt} = 0,\quad i = 1, \dots, n,$$

Eqs (22)–(28) have the following solution for $r = 0$

$$X_{\mathrm{i}}^{(0)} = X_{0}^{(0)} = 1 \tag{30}$$

$$Y_{\mathrm{i}}^{(0)} = mX_{0}^{(0)} = m\,. \tag{31}$$

By summing Eqs (22), (24) and (26) and after substituting for $X_0^{(\mathrm{r})}$ from Eq. (29), we get

$$-Tr\sum_{i=1}^{n} X_{\mathrm{i}}^{(\mathrm{r}-1)} = \frac{N_{\mathrm{x}}}{m}\sum_{i=1}^{n} Y_{\mathrm{i}}^{(\mathrm{r})} - N_{\mathrm{x}}\sum_{i=1}^{n} X_{\mathrm{i}}^{(\mathrm{r})} - X_{\mathrm{n}}^{(\mathrm{r})}\,, \quad r \geqq 1\,. \tag{32}$$

By summing Eqs (23), (25) and (27), we obtain

$$-Tr\frac{E}{m}\sum_{i=1}^{n} Y_{\mathrm{i}}^{(\mathrm{r}-1)} = N_{\mathrm{x}}\sum_{i=1}^{n} X_{\mathrm{i}}^{(\mathrm{r})} - \frac{N_{\mathrm{x}}}{m}\sum_{i=1}^{n} Y_{\mathrm{i}}^{(\mathrm{r})}\,, \quad r \geqq 1\,. \tag{33}$$

And, finally, by summing Eqs (32) and (33), we obtain the relation

$$X_{\mathrm{n}}^{(\mathrm{r})} = Tr\left(\sum_{i=1}^{n} X_{\mathrm{i}}^{(\mathrm{r}-1)} + \frac{E}{m}\sum_{i=1}^{n} Y_{\mathrm{i}}^{(\mathrm{r}-1)}\right), \quad r \geqq 1 \tag{34}$$

which makes it possible to calculate the r-th moment of normalized response in the n-th stage from the known $(r - 1)$-th moments of normalized responses in all stages.

Mean and Variance of Normalized Response in n-th Stage

By inserting the zeroth moments from Eqs (28), (30) and (31) into Eq. (34) for $r = 1$ and on rearranging, we obtain the relation for the first moment (mean), $\mu = X_{\mathrm{n}}^{(1)}$, of the response in the n-th stage:

$$\mu = nT(1 + E)\,. \tag{35}$$

The dimensionless variance σ^2/μ^2 of response in the n-th stage is defined by

$$\frac{\sigma^2}{\mu^2} = \frac{X_{\mathrm{n}}^{(2)}}{\mu^2} - 1\,. \tag{36}$$

After inserting $\sum_{i=1}^{n} Y_{\mathrm{i}}^{(1)}$ from Eq. (33) for $r = 1$ and $Y_{\mathrm{i}}^{(0)}$ from Eq. (31) into Eq. (34) for $r = 2$, we get

$$X_{\mathrm{n}}^{(2)} = 2T(1 + E)\sum_{i=1}^{n} X_{\mathrm{i}}^{(1)} + \frac{2nT^2E^2}{N_{\mathrm{x}}}\,. \tag{37}$$

Therefore, the knowledge of the first moments in all stages is necessary for calculating the variance. By combining Eqs (*24*) and (*25*) for $r = 1$, with (*30*) and (*31*) (see (*D1*)), we obtain the linear difference equation with constant coefficients for $X_i^{(1)}$:

$$\frac{abm}{N_x} X_{i+2}^{(1)} - \left[bm + a + \frac{bm(1 + 4a)}{N_x}\right] X_{i+1}^{(1)} +$$
$$+\left[2(bm + a) + 1 + \frac{3bm(1 + 2a)}{N_x}\right] X_i^{(1)} - \left[bm + a + 1 + \frac{bm(3 + 4a)}{N_x}\right] X_{i-1}^{(1)} +$$
$$+ \frac{bm(1 + a)}{N_x} X_{i-2}^{(1)} = T(1 + E), \quad i = 3, \dots n - 2. \tag{38}$$

Non-homogeneous difference equation (*38*) has the general solution

$$X_i^{(1)} = \bar{X}_i^{(1)} + \tilde{X}_i^{(1)}, \tag{39}$$

where $\bar{X}_i^{(1)}$ is the general solution of homogeneous equation and $\tilde{X}_i^{(1)}$ is the particular solution of non-homogeneous equation. The characteristic equation of homogeneous form of difference equation (*38*) can be written in the form

$$(\lambda - 1)\left\{\frac{abm}{N_x}\lambda^3 - \left[bm + a + \frac{bm(1 + 3a)}{N_x}\right]\lambda^2 +\right.$$
$$\left. + \left[bm + a + 1 + \frac{bm(2 + 3a)}{N_x}\right]\lambda - \frac{bm(1 + a)}{N_x}\right\} = 0. \tag{40}$$

It follows from Eq. (*40*) and theorem (*D2*) that characteristic equation (*40*) has one root equal to one:

$$\lambda_1 = 1, \tag{41}$$

and the remaining three roots are positive, real and different.

If we take account of the validity of Eq. (*41*), we can write the general solution of homogeneous equation in the form

$$\bar{X}_i^{(1)} = P_1 + P_2\lambda_2^i + P_3\lambda_3^i + P_4\lambda_4^i, \tag{42}$$

where P_j, $j = 1, \dots, 4$ are constants which can be obtained from boundary conditions. Let the particular solution have the form

$$\tilde{X}_i^{(1)} = iR, \tag{43}$$

where R is a constant. On inserting this solution for $X_i^{(1)}$ into Eq. (38) and taking account of the validity of Eq. (40), we get, after rearranging, the relation for R

$$R = T(1 + E) . \tag{44}$$

The general solution of difference equation (38) has then the form

$$X_i^{(1)} = P_1 + \sum_{j=2}^{4} P_j \lambda_j^i + T(1 + E)\, i . \tag{45}$$

After inserting $X_i^{(0)}$ and $X_0^{(1)}$ from conditions (30) and (29) into Eqs (22)–(27) for $r = 1$, $i = 1, 2, n - 1, n$ and further on using a similar procedure which has led to Eq. (38), we obtain Eqs (D3) in Appendix. By inserting $X_i^{(1)}$ from Eq. (45) into these equations and solving for P_j taking account of validity of Eq. (40), we get the equations

$$P_j = T(1 + E)\, C_j , \quad j = 1, \dots 4 , \tag{46}$$

where

$$C_1 = bm + a , \tag{47}$$

$$\sum_{j=2}^{4} C_j[a(\lambda_j - 1) - 1] = bm , \tag{48}$$

$$\sum_{j=2}^{4} C_j \lambda_j^n = -(bm + a) , \tag{49}$$

$$\sum_{j=2}^{4} C_j \lambda_j^{n+1} = -(bm + a + 1) . \tag{50}$$

It follows from Eqs (45) and (46) that the general solution of difference equation can be written in terms of constants C_j, $j = 1, \dots, 4$ in the form

$$X_i^{(1)} = T(1 + E)\left[C_1 + \sum_{j=2}^{4} C_j \lambda_j^i + i\right] . \tag{51}$$

By solving Eqs (48)–(50) for C_2, C_3, C_4, we get the relation

$$\begin{aligned} C_j = \{&[(a + bm)(\lambda_k - 1) - 1][a(\lambda_l - 1) - 1]\lambda_k^n - \\ &- [(a + bm)(\lambda_l - 1) - 1][a(\lambda_k - 1) - 1]\lambda_l^n + bm(\lambda_k - \lambda_l)\lambda_k^n \lambda_l^n\}/ \\ &/\{\lambda_2^n \lambda_3^n (\lambda_2 - \lambda_3)[a(\lambda_4 - 1) - 1] + \lambda_2^n \lambda_4^n (\lambda_4 - \lambda_2)[a(\lambda_3 - 1) - 1] + \\ &+ \lambda_3^n \lambda_4^n (\lambda_3 - \lambda_4)[a(\lambda_2 - 1) - 1]\} , \end{aligned} \tag{52}$$

where subscripts j, k, l take gradually the values

$$k = 3\,, \quad l = 4 \quad \text{for} \quad j = 2\,;$$

$$k = 4\,, \quad l = 2 \quad \text{for} \quad j = 3\,;$$

$$k = 2\,, \quad l = 3 \quad \text{for} \quad j = 4\,.$$

After inserting C_1 from Eq. (*47*) into (*51*), it is possible to carry out the summation $\sum_{i=1}^{n} X_i^{(1)}$ in Eq. (*37*) which is given in Appendix as Eq. (*D4*). By combining Eqs (*35*)–(*37*) with the expression for $\sum_{i=1}^{n} X_i^{(1)}$, we obtain the resulting relation for dimensionless variance

$$\frac{\sigma^2}{\mu^2} = \frac{1}{n}\left[1 + 2\,(bm + a) + \frac{2}{N_x}\left(\frac{E}{1 + E}\right)^2\right] + \frac{2}{n^2}\sum_{j=2}^{4} C_j(\lambda_j^n - 1)\frac{\lambda_j}{\lambda_j - 1}\,, \qquad (53)$$

where Eq. (*52*) holds for C_j and Eq. (*40*) for roots λ_j.

Special Cases

Practically interesting simplifications are those ones when the longitudial mixing in one phase is negligible ($a \to 0$, and/or $b \to 0$), mass transfer between phases does not take place ($m = 0$), mass transfer between phases is instantaneous ($N_x \to \infty$) and the number of stages is large ($n \gg 1$).

In the simplified cases, the decrease in the degree of characteristic equation occurs, and one or two roots disappears. The calculation of constants C_j from Eq. (*52*) requires, however, information on the magnitude of these roots.

Let us denote the roots corresponding to the case $a \to 0$ by symbols λ_2, λ_3 and the root corresponding to the case $b \to 0$ by symbol λ_4. From the analysis of the root magnitude for $a \to 0$, $b \to 0$ follows:

For roots of characteristic equation (*40*) holds

$$\lambda_2 > 1\,, \quad \lambda_3 < 1\,, \quad \lambda_4 > 1\,; \qquad (54)$$

with the simplification of general case to the case

$$a \to 0 \quad \text{root} \quad \lambda_4 \to \infty\,, \qquad (55)$$

$$b \to 0 \quad \text{root} \quad \lambda_2 \to \infty\,, \quad \lambda_3 \to 0\,. \qquad (56)$$

From inequality (*54*), it follows for the case $n \gg 1$

$$\lambda_2^n \to \infty\,, \quad \lambda_3^n \to 0\,, \quad \lambda_4^n \to \infty\,. \tag{57}$$

For the simplified cases given below, information (*54*)–(*56*) was used when calculating constants C_j from Eq. (*52*). After inserting constants C_j and roots λ_j into Eq. (*53*), the resulting expression for dimensionless variance was then obtained.

First let us consider the case when the longitudial mixing in the dispersed phase is negligible, $a \to 0$. Then for the mean of response holds Eq. (*35*), for the characteristic equation and its roots the equations

$$bm(1 + N_x)\,\lambda^2 - [bm(2 + N_x) + N_x]\,\lambda + bm = 0 \tag{58}$$

$$\lambda_{2,3} = \frac{2bm + N_x(bm + 1) \pm \{N_x(bm + 1)]^2 + 4bm\}^{1/2}}{2bm(1 + N_x)} \tag{59}$$

and for the dimensionless variance the expression

$$\frac{\sigma^2}{\mu^2} = \frac{1}{n}\left[1 + 2bm + \frac{2}{N_x}\left(\frac{E}{1 + E}\right)^2\right] - \\ - \frac{2}{n^2}\,bm\,\frac{(\lambda_3^n - 1)(\lambda_2^n - 1)}{\lambda_3^n - \lambda_2^n}\left(\frac{\lambda_2}{\lambda_2 - 1} - \frac{\lambda_3}{\lambda_3 - 1}\right). \tag{60}$$

In case of $b \to 0$, Eq. (*35*) holds for the mean of response, for the characteristic equation and its root λ_4 hold the relations

$$-a\lambda_4 + a + 1 = 0 \tag{61}$$

$$\lambda_4 = \frac{a + 1}{a} \tag{62}$$

and for the dimensionless variance of response the expression

$$\frac{\sigma^2}{\mu^2} = \frac{1}{n}\left[1 + 2a + \frac{2}{N_x}\left(\frac{E}{1 + E}\right)^2\right] - \frac{2}{n^2}\,a(a + 1)\left[1 - \left(\frac{a}{a + 1}\right)^n\right]. \tag{63}$$

If the instantaneous mass transfer takes place between phases, $N_x \to \infty$, then Eq. (*35*) holds for the mean of response, for the characteristic equation and its root hold the equations

$$-(a + bm)\,\lambda + a + bm + 1 = 0 \tag{64}$$

$$\lambda = \frac{a + bm + 1}{a + bm} \tag{65}$$

and for the variance the expression

$$\frac{\sigma^2}{\mu^2} = \frac{1}{n} [1 + 2(a + bm)] - \frac{2}{n^2}(a + bm)(a + bm + 1)\left[1 - \left(\frac{a + bm}{a + bm + 1}\right)^n\right]. \tag{66}$$

If the injection into the dispersed phase is formed by a substance which does not pass to the continuous phase ($m = 0$ and consequently $E = 0$, too),

$$\mu = nT = n\Phi \frac{V}{Q_x} \tag{67}$$

holds for the mean of response,

$$-a\lambda + a + 1 = 0 \tag{68}$$

$$\lambda = \frac{a + 1}{a} \tag{69}$$

for the characteristic equation and its root, and

$$\frac{\sigma^2}{\mu^2} = \frac{1}{n}(1 + 2a) - \frac{2}{n^2} a(a + 1)\left[1 - \left(\frac{a}{a + 1}\right)^n\right] \tag{70}$$

for the dimensionless variance.

The resulting equations can be approximated, for a great number of stages (i.e., with the validity of Eq. (57)), by the relations which are summarized in Table I.

RESULTS AND DISCUSSION

The analytical solution of linear form of the model with back flows results in non-linear algebraic dependences between the first two moments of concentration responses and model parameters Φ, a, b, N_x.

The derivation of linear form of the model is based on several assumptions. The condition of constant flow rate of phase is fulfilled, e.g., in a wide class of problems concerning the extraction of metals with chemical reaction. If the extraction takes place in the region of low concentrations of solute, it is often possible, even in case of extraction with chemical reaction, to consider the distribution coefficient and the mass transfer coefficient to be constant. For a normal coalescing system and a column of usual length, the assumption of invariability of the size of interfacial area with the column length is usually fulfilled to a sufficient extent.

In the linear case, it is possible to calculate, by means of the derived relations, the following quantities:

- hold-up of the dispersed phase, Φ, from the first moment of response to the δ-injection of tracer into the dispersed phase (Eq. (*67*));
- back-flow coefficient in the dispersed phase, a, from the dimensionless variance of response to δ-injection of tracer into the dispersed phase (Eq. (*70*));
- mass transfer number, N_x, from the dimensionless variance of response to the δ-injection of solute into the dispersed phase (Eqs (*53*, (*52*) and (*40*)) since the

TABLE I

Resulting expressions for mean and dimensionless variance of concentration response for a great number of stages

Simplification	μ	$\dfrac{\sigma^2}{\mu^2}$
–	$nT(1+E)$	$\dfrac{1}{n}\left[1 + 2(bm+a) + \dfrac{2}{N_x}\left(\dfrac{E}{1+E}\right)^2\right] - \dfrac{2}{n^2}\left[\dfrac{(bm+a)(\lambda_2\lambda_4 - 1) - 1}{(\lambda_2 - 1)(\lambda_4 - 1)} + \dfrac{bm}{a(\lambda_3 - 1) - 1}\dfrac{\lambda_3}{\lambda_3 - 1}\right]$, $\lambda_2, \lambda_4 > 1; \lambda_3 < 1$; where for $\lambda_2, \lambda_3, \lambda_4$ is: $\dfrac{abm}{N_x}\lambda^3 - \left[bm + a + \dfrac{bm(1+3a)}{N_x}\right]\lambda^2 + \left[bm + a + 1 + \dfrac{bm(2+3a)}{N_x}\right]\lambda - \dfrac{bm(1+a)}{N_x} = 0$
$m = 0$	nT	$\dfrac{1}{n}[1 + 2a] - \dfrac{2}{n^2}a(a+1)$
$N_x \to \infty$	$nT(1+E)$	$\dfrac{1}{n}[1 + 2(bm+a)] - \dfrac{2}{n^2}[a(a+1) + bm(bm+1) + 2abm]$
$b = 0$	$nT(1+E)$	$\dfrac{1}{n}\left[1 + 2a + \dfrac{2}{N_x}\left(\dfrac{E}{1+E}\right)^2\right] - \dfrac{2}{n^2}a(a+1)$
$a = 0$	$nT(1+E)$	$\dfrac{1}{n}\left[1 + 2bm + \dfrac{2}{N_x}\left(\dfrac{E}{1+E}\right)^2\right] - \dfrac{2}{n^2}bm\left[(bm+1)^2 + \dfrac{4bm}{N_x}\right]^{1/2}$

missing back-flow coefficient in the continuous phase, b, can be obtained, e.g., by the method reported by Pekovich et al.[2]

In the nonlinear case, it is possible to obtain a good initial estimate of parameters Φ, a, N_x from the approximate expressions given in Table I. Neglecting the last term in the expression for dimensionless variance, which is possible in usual case of sufficiently long columns, results in a great simplification of calculating the estimate of parameter N_x.

Knowledge of these parameters makes it possible to predict the concentration profile and thus the column efficiency by using the method reported in the preceding work[1].

Relation (35) for the mean of the response to the δ-injection of solute can serve for checking the correctness of extraction ratio.

It follows from Eq. (53) that the dependence of dimensionless variance σ^2/μ^2 on the reciprocal value of mass transfer number $1/N_x$ approaches a linear dependence with increasing number of stages. We can see in Fig. 2 that the sensitivity of variance σ^2/μ^2 to $1/N_x$ increases with increasing value of E, i.e., with decreasing hold-up of the dispersed phase.

CONCLUSION

The method of predicting the efficiency of mechanically agitated extraction columns[1] stems from the concentration responses (i.e., dependences of concentrations on time) to δ-injection of observed substances into the equipment which operate with stationary continuous phase. The responses are evaluated in terms of a mathematical model. In a general case, the parameters of model are obtained by the numerical solution of differential equations combined with the method of fitting the response curves or their moments. Then operating conditions are chosen so that it is possible to

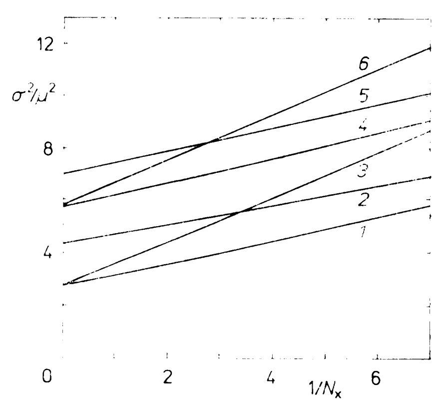

FIG. 2

Dependence of dimensionless variance of response on reciprocal value of mass transfer number, $n = 20$. *1* $a = 0$, $bm = 1$, $E = 1$; *2* $a = 0$, $bm = 2$, $E = 1$; *3* $a = 0$, $bm = 1$, $E = 2$; *4* $a = 2$, $bm = 1$, $E = 1$; *5* $a = 2$, $bm = 2$, $E = 1$; *6* $a = 2$, $bm = 1$, $E = 2$

expect the validity of parameters obtained even in countercurrent arrangement, and by solving the model, the concentration profiles and the column efficiency are calculated.

The analytical solution of linear cases leads to the algebraic relations between the model parameters and moments of responses. The model with back flow in the continuous and short cut in the dispersed phase was solved by Wichterlová, Rod and Hančil[1].

The analytical solution of linear form of stagewise model with back flows in both phases is presented in this paper. The relations between moments (mean and dimensionless variance) of concentration responses and parameters (hold-up of the dispersed phase, coefficient of back flow in the dispersed phase, and volumetric mass transfer coefficient) of the model were derived.

In the linear case, the resulting expressions (*67*), (*70*), (*53*), (*52*) and (*40*) can serve for calculating the parameters, in the nonlinear case, the approximate form of these relations (Table I) is suitable for determining the initial guesses of parameters for an iterative procedure of calculating their values.

APPENDIX

(*D1*) From Eq. (*24*), $r = 1$, follows for $i = 2, \ldots, n - 1$

$$Y_i^{(1)} = \frac{m}{N_x}\left[(1 + N_x) X_i^{(1)} - X_{i-1}^{(1)} - a(X_{i+1}^{(1)} - 2X_i^{(1)} + X_{i-1}^{(1)}) - T\right],$$

analogously holds for $i = 3, \ldots, n - 2$

$$Y_{i-1}^{(1)} = \frac{m}{N_x}\left[(1 + N_x) X_{i-1}^{(1)} - X_{i-2}^{(1)} - a(X_i^{(1)} - 2X_{i-1}^{(1)} + X_{i-2}^{(1)}) - T\right]$$

$$Y_{i+1}^{(1)} = \frac{m}{N_x}\left[(1 + N_x) X_{i+1}^{(1)} - X_i^{(1)} - a(X_{i+2}^{(1)} - 2X_{i+1}^{(1)} + X_i^{(1)}) - T\right].$$

(*D2*) It is known that the characteristic polynomial has at most so many real positive roots, how many are sign changes in the series of coefficients of the polynomial. If these roots are fewer, they are even number fewer.

(*D3*) $i = 1$:

$$T(1 + E) = \frac{abm}{N_x} X_3^{(1)} - \left[bm + a + \frac{bm(1 + 3a)}{N_x}\right] X_2^{(1)} + \\ + \left[bm + a + 1 + \frac{2bm(1 + a)}{N_x}\right] X_1^{(1)} .$$

$i = 2$:

$$T(1 + E) = \frac{abm}{N_x} X_4^{(1)} - \left[bm + a + \frac{bm(1 + 4a)}{N_x}\right] X_3^{(1)} +$$

$$+ \left[2(bm + a) + 1 + \frac{3bm(1 + 2a)}{N_x}\right] X_2^{(1)} - \left[bm + a + 1 + \frac{3bm(1 + a)}{N_x}\right] X_1^{(1)}$$

$i = n - 1$:

$$T(1 + E) = -\left[bm + a + \frac{bm(1 + 3a)}{N_x}\right] X_n^{(1)} +$$

$$+ \left[2(bm + a) + 1 + \frac{3bm(1 + 2a)}{N_x}\right] X_{n-1}^{(1)} -$$

$$- \left[bm + a + 1 + \frac{bm(3 + 4a)}{N_x}\right] X_{n-2}^{(1)} + \frac{bm(1 + a)}{N_x} X_{n-3}^{(1)}$$

$i = n$:

$$T(1 + E) = \left[bm + a + 1 + \frac{bm(1 + 2a)}{N_x}\right] X_n^{(1)} -$$

$$- \left[bm + a + 1 + \frac{bm(2 + 3a)}{N_x}\right] X_{n-1}^{(1)} + \frac{bm(1 + a)}{N_x} X_{n-2}^{(1)}$$

$$\sum_{i=1}^{n} X_i^{(1)} = nT(1 + E)\left[a + bm + \frac{1}{n} \sum_{j=2}^{4} C_j \lambda_j \frac{\lambda_j^n - 1}{\lambda_j - 1} + \frac{1 + n}{2}\right] \tag{D4}$$

SYMBOLS

a	back-flow coefficient in dispersed phase, Eq. (*13*)
A	interfacial area, m^2
b	back-flow coefficient in continuous phase, Eq. (*13*)
E	extraction ratio, Eq. (*11*)
J	interfacial flux, kmol m^{-2} s^{-1}
k_x	mass transfer coefficient, m s^{-1}
m	equilibrium distribution coefficient, $m = (y/x)_{eq}$
M	injected solute amount, kmol
n	number of stages
N_x	mass transfer number, Eq. (*12*)
q	back flow, m^3 s^{-1}
Q	flow rate, m^3 s^{-1}
t	time, s
T	mean residence time of dispersed phase in stage, Eq. (*10*), s
v	volume of phase in stage, m^3
V	stage volume, m^3

x	concentration in dispersed phase, kmol m^{-3}
$X^{(r)}$	r-th moment of normalized response in dispersed phase
y	concentration in continuous phase, kmol m^{-3}
$Y^{(r)}$	r-th moment of normalized response in continuous phase
Φ	hold-up of dispersed phase
λ	root of characteristic equation
μ	mean residence time of solute in column, s
σ^2	variance of residence time, s^2

Subscripts

i	i-th stage
x	dispersed phase
y	continuous phase

REFERENCES

1. Wichterlová J., Rod V., Hančil V.: Chem. Eng. Process. *29*, 39 (1991).
2. Pekovich L., Pebalk V. L.: Teor. Osn. Khim. Tekhnol. *4*, 267 (1970).

Translated by J. Linek.